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Triple Domination Number and Its Connectivity of Complete Graphs

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ABSTRACT
In a graph G, a vertex dominates itself and its neighbours. A subset S of V is called a dominating set in G if every vertex in V is dominated by at least one vertex in S. The domination number γ (G) is the minimum cardinality of a dominating set. A set $S \subseteq V$ is called a Triple dominating set of a graph G if every vertex in V dominated by at least three vertices in S. The minimum cardinality of a triple dominating set is called Triple domination number of G and is denoted by Tγ(G). The connectivity κ (G) of a connected graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph. In this paper we find an upper bound for the sum of the Triple domination number and connectivity of a graph and characterize the corresponding extremal graphs.
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Key Words: Domination, Domination number, Triple domination, Triple domination number and connectivity.

I. INTRODUCTION

The concept of domination in graphs evolved from a chess board problem known as the Queen problem- to find the minimum number of queens needed on an 8x8 chess board such that each square is either occupied or attacked by a queen. C.Berge[12] in 1958 and 1962 and O.Ore[11] in 1962 started the formal study on the theory of dominating sets. Thereafter several studies have been dedicated in obtaining variations of the concept. The authors in [2] listed over 1200 papers related to domination in graphs in over 75 variation.

The graph G = (V, E) we mean a finite , undirected and connected graph with neither loops nor multiple edges. The order and size of G are denoted by n and m respectively. The degree of any vertex u in G is the number of edges incident with u is denoted by d(u). The minimum and maximum degree of a graph G is denoted by δ(G) and $\Delta(G)$ respectively. For graph theoretic terminology we refer to Chartrand and Lesniak[1] and Haynes et.al[2,3].

Let v∈V. The open neighbourhood and closed neighbourhood of v are denoted by N(v) and N[v] = N(v)∪{v} respectively. If $S\subseteq V$ then N(S) $=\bigcup_{v\in S} N(v)$ for all v∈S and N[S] = N(S) ∪S. If $S\subseteq V$ and u∈S then the private neighbour set of u with respect to S is defined by pn[u , S] = {v : N[v]∩S ={u} }. We denote a cycle on n vertices by C_n, a path on n vertices by P_n and a complete graph on n vertices by K_n. A bipartite graph is a graph whose vertex set can be divided into two disjoint sets V_1 and V_2 such that every edge has one end in V_1 and another in V_2. A complete bipartite graph is a bipartite graph where every vertex of V_1 is adjacent to every vertex in V_2. The complete bipartite graph with partitions of order $|v_1| = m$, and $|v_2| = n$ is denoted by $K_{m,n}$. A wheel graph , denoted by W_n is a graph with n vertices formed by connecting a single vertex to all vertices of C_{n-1}. H{m_1, m_2,........,m_n) denotes the graph obtained from the graph H by pasting m_i edges to the vertex v_i∈V(H), 1 ≤ i ≤ n , H(P_{m1}, P_{m2}…..,P_{mn}) is the graph obtained from the graph H by attaching the end vertex of P_{mi} to the vertex v_i in H, 1 ≤ i ≤ n. Bistar B(r ,s) is a graph obtained from $K_{1,r}$ and $K_{1,s}$ by joining its centre vertices by an edge.

In a graph G , a vertex dominates itself and its neighbours. A subset S of V is called a dominating set in G if every vertex in V is dominated by at least one vertex in S. The domination number γ(G) is the minimum cardinality of a dominating set. Harary and Haynes[4] introduced the concept of double domination in graphs. A set $S\subseteq V$ is called a double dominating set of a graph G if every vertex in V is dominated by at least two vertices in S The minimum cardinality of double dominating set is called double domination number of G and is denoted by dd(G). A vertex cut, or separating set of a connected graph G is a set of vertices whose removal results in a disconnected graph. The connectivity or vertex connectivity of a graph G denoted by κ(G) . The Connectivity κ(G) of a connected graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph. A matching M is a subset of edges so that every vertex has degree at most one in M.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. J.Paulraj Joseph and S. Arumugam[5] proved that $\gamma(G) + \kappa(G) \leq n$ and characterized the corresponding extremal graphs . In this paper , we obtained an upper bound for the sum of the Triple domination number and connectivity of a graph characterized the corresponding extremal graphs. We use the following theorems.

Theorem 1.1. [2] For any graph G, $dd(G) \leq n$

Theorem 1.2. [1] For a graph G, $\kappa(G) \leq \delta(G)$

II. MAIN RESULTS

Definition 2.1

A set $S \subseteq V$ is called *Triple dominating set* of a graph G . If every vertex in V is dominated by at least three vertices in S. The minimum cardinality of Triple dominating set is called *Triple domination number* of G and is denoted by $T\gamma(G)$. The connectivity $\kappa(G)$ of a connected graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph.

Choose $S=\{v_1,v_2,v_3\} \in V(G)$, If $N[S] = V(G)$, A dominating set obtained in the way given above is called a Triple dominating set.

Example 2.2. For the graph K_3, $T\gamma(G) = 3$, and $\kappa(G) = 2$.We have $T\gamma(G) + \kappa(G) = 5$

Example 2.3. For the graph K_4, $T\gamma(G) = 3$, and $\kappa(G) = 3$. We have $T\gamma(G) + \kappa(G) = 6$

Example 2.4. For the graph K_5, $T\gamma(G) = 3$, and $\kappa(G) = 4$. We have $T\gamma(G) + \kappa(G) = 7$

Example 2.5. For the graph K_6, $T\gamma(G) = 3$, and $\kappa(G) = 5$. We have $T\gamma(G) + \kappa(G) = 8$

Lemma 2.6. For the complete graph K_n, We have (i)$T\gamma(G) + \kappa(G) = 2n-1$ when n=3 (ii)$T\gamma(G) + \kappa(G) \leq 2n -1$ when n=4 (iii)$T\gamma(G) + \kappa(G) \leq 2n -2$ when n=5 (iv)$T\gamma(G) + \kappa(G) \leq 2n -3$ when n=6.

Result 2.7.[14] For any graph K_n , We have $\gamma(G) \leq \gamma_N(G) \leq T\gamma(G)$ .Where $\gamma_N(G)$ is the degree equitable domination number of G.

Lemma 2.8. For any graph G, $T\gamma(G) \leq n$.

Lemma 2.9. For any connected graph G, $\kappa(G) = n-1$ if G is isomorphic to K_n.

Result 2.10. For any connected graph G , $T\gamma(G) = 3$ if G is isomorphic to K_n , $n \geq 3$

Result 2.11. For the graph $K_{m,n}$ where m=n.There exists a Triple dominating set with matching M

Theorem 2.12. Let G_1 and G_2 be any two graph of Triple dominating sets then $G_1 + G_2$ is a graph of Triple dominating set of G_1 or G_2.

Proof : Let G_1 and G_2 be any two graphs having triple dominating sets .By taking sum of G_1 and G_2 , we have every vertex in G_1 is adjacent to every vertex in G_2. Therefore by the definition of triple dominating set , we have By choosing S is the Triple dominating set of G_1 or G_2 and $N[S] = V(G_1 + G_2)$. Hence S is the Triple dominating set of G_1 or G_2.

Theorem 2.13. Every complete graph K_n has a Triple dominating set if $n \geq 3$.

Proof : Given the graph G is complete when $n \geq 3$, Choose $S =\{v_1,v_2,v_3\} \in V(G)$, If $N[S] = V(G)$, A dominating set obtained is a $T\gamma(G) = 3$.

Theorem 2.14. For any connected graph G, $T\gamma(G) + \kappa(G) = 2n -1$ if and only if G is isomorphic to K_3.

Proof :

Case 1. $T\gamma(G) + \kappa(G) \leq n + \delta \leq n + n -1= 2n -1$. Let $T\gamma(G) + \kappa(G) = 2n -1$ then $T\gamma(G) = n$ and $\kappa(G) = n -1$. Then G is a complete graph on n vertices. Since $T\gamma(K_n) = 3$ we have $n = 3$. Hence G is isomorphic to K_3. The converse is obvious.

Case 2. Suppose $T\gamma(G) = n-1$ and $\kappa(G) = n$ then $n \leq \delta(G)$ is impossible which is a contradiction to $\kappa(G) = n-1$.Hence $T\gamma(G) = n-1$ and $\kappa(G) = n$ is not possible.

Theorem 2.15 . For any connected graph G, $T\gamma(G) + \kappa(G) = 2n -2$ if and only if G is isomorphic to K_4 or C_4 .

Proof: $T\gamma(G) + \kappa(G) = 2n -2$, then there are two cases to be considered.

(i) $T\gamma(G) = n-1$ and $\kappa(G) = n -1$ (ii) $T\gamma(G) = n$ and $\kappa(G) = n - 2$

Proof :

Case 1. $T\gamma(G) = n-1$ and $\kappa(G) = n -1$, Then G is a complete graph on n vertices, Since $T\gamma(G) = 3$, We have $n = 4$. Hence G is isomorphic to K_4.

Case 2. $T\gamma(G) = n$ and $\kappa(G) = n - 2$, Then $n - 2 \leq \delta(G)$. If $\delta = n -1$, Then G is a complete graph , Which is a contradiction . Hence $\delta(G) = n - 2$. Then G is isomorphic to $K_n - M$ where M is a matching in K_n. Then $T\gamma(G) = 3$ or 4, If $T\gamma(G) = 3$ then $n = 3$. Which is a contradiction to $\kappa(G) =1 \neq n-2$ and $N[S] \neq V(G)$, Thus $T\gamma(G) = 4$. Then $n = 4$ and hence G is isomorphic toK_4-e or C_4 with $|M|$=1 or 2 respectively. Given fig:1a. and fig:1b.

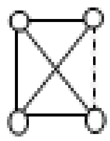
fig:1a

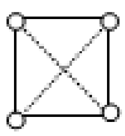
fig:1b

Theorem 2.16. For any connected graph G , $T\gamma(G) + \kappa(G) = 2n -3$ if and only if G is isomorphic to K_4 or $K_4 - e$ or $K_{1,3}$ or $K_5 - M$, Where M is a matching on K_5 with $|M| = 1$.

Proof : Let $T\gamma(G) + \kappa(G) = 2n - 3$, Then there are three cases to be considered (i) $T\gamma(G) = n-2$ and $\kappa(G) = n -1$ (ii) $T\gamma(G) = n- 1$ and $\kappa(G) = n - 2$ (iii) $T\gamma(G) = n$ and $\kappa(G) = n - 3$.

Case 1 .

Tγ(G) = n-2 and κ(G) = n -1, Then G is a complete graph on n vertices, Since Tγ(G) = 3 and $\delta(G)$= n is not possible ,We have n = 5. Hence G is isomorphic to K_5.

Case 2 .

Tγ(G) = n- 1 and κ(G) = n – 2, Then n - 2≤ δ (G) : If δ = n -1 then G is a complete graph, which gives a contradiction to κ(G) = n – 2. If δ (G) = n -2, Then G is isomorphic to K_n – M where M is a matching in K_n then Tγ(G) = 3 or 4. If Tγ(G) = 3 then n = 4 , Then G is either C_4 or K_4 –e , But Tγ(G) = 4 ≠ n – 1. Hence G is isomorphic to K_4 –e where 'e' is a matching in K_4 by fig:2a. If Tγ(G) = 4 then n = 5 and hence G is isomorphic to K_5- M where M is a matching on K_5 with $|M| = 1$ by Fig:2b.

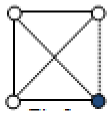
Fig:2a

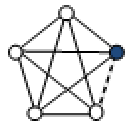
Fig:2b

Case 3 .

Tγ(G) = n and κ(G) = n – 3 then n - 3≤ δ (G) . If δ = n -1 then G is a complete graph, which gives a contradiction to κ(G) = n – 3. If δ (G) = n -2, Then G is isomorphic to K_n – M where M is a matching in K_n then Tγ(G) = 3 or 4. Then n = 3 or 4, Since n = 3 is impossible, We have n= 4, Then G is either K_4 –e or C_4 . For these two graphs κ(G) ≠ n -3 which is a contradiction. Hence δ = n -3.
Let X be the vertex cut of G with $|X|$= n -3 and let V – X = { x_1 ,x_2 ,x_3}, X = {v_1,v_2,...........v_{n-3}}.

Sub Case 3.1. $\langle V-X\rangle = \overline{K_3}$
Then every vertex of V – X is adjacent to all the vertices of X. Then {(V- X) ∪ $\{v_1\}$ } – M is a Triple dominating set of G where M is a matching in K_n and hence Tγ(G) ≤ 4, This gives n ≤ 5, Since n ≤ 3 is impossible , we have n = 4 or 5. If n = 4 then G is isomorphic to $K_{1,3}$ which is a contradiction to the definition of matching by fig:2c. If n = 5 then the graph G has Tγ(G) = 3 or 4 which is a contradiction to κ(G) = n – 3 by fig:2d.

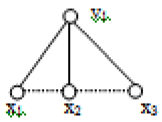

fig:2c

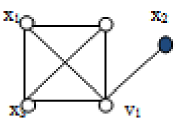

fig:2d

Sub case 3.2. $\langle V-X\rangle = K_1 \cup K_2$

Let x_1x_2ЄE(G), Then x_3 is adjacent to all the vertices in X and x_1, x_2 are not adjacent to at most one vertex in X, If v_1∉ N(x_1) ∪N(x_2) then {(V- X) ∪ $\{v_1\}$ } is a triple dominating set of G and hence Tγ(G) ≤ 4 ,This gives n = 4. For this graph κ(G) = 1 which is a contradiction to κ(G)=n-3 by fig:3a . So all v_i , either v_iЄN(x_1) or v_iЄN(x_2) or both, Then {(V- X) ∪ $\{v_i\}$ } is a triple dominating set of G. Hence Tγ(G) ≤ 4, then n = 4. Hence G is isomorphic to C_4 or k_3(1 ,0, 0). But Tγ[k_3(1 ,0, 0)] = 3 ≠ n . Which is a contradiction by fig 3b. The converse is obvious.

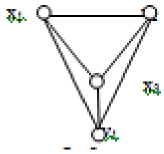
Fig 3a

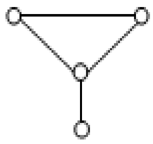
fig:3b

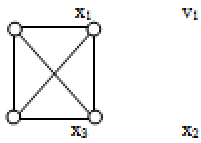

fig:3c

Theorem 2.17. For any connected graph G, Tγ(G) + κ(G) = 2n – 4 if and only if G is isomorphic to K_6 or K_4-e or $K_{1,4}$ or K_3(1,0,0) or B(1,2) or K_5-M where M is a matching on K_5 with $|M| = 1$.

Proof. Let Tγ(G) + κ(G) = 2n – 4. Here there are four cases to be discussed (i) Tγ(G) = n – 3 and κ(G) = n – 1 (ii) Tγ(G) = n – 2 and κ(G) = n – 2 (iii) Tγ(G) = n – 1 and κ(G) = n – 3 (iv) Tγ(G) = n and κ(G) = n – 4

Case 1. Tγ(G) = n – 3 and κ(G) = n – 1 , Then G is a complete graph on n vertices ,Since Tγ(G)=3 we have n = 6 . Hence G is isomorphic to K_6.

Case 2. Tγ(G) = n – 2 and κ(G) = n – 2 then n-2≤ δ. If $\delta = n-1$ then G is a complete graph which is a contradiction. If $\delta = n-2$, Then G is isomorphic to K_n-M is a matching in K_n, then Tγ(G)= 3 or 4. If Tγ(G)=3 then n=5 then G is either C_5 or K_5-e by fig:4a. If Tγ(G)=4 then n=6 and hence G is isomorphic to K_6-e by fig:4b.

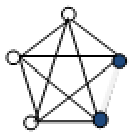
fig:4a

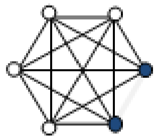
fig:4b

Case 3.Tγ(G) = n – 1 and κ(G) = n – 3, Then n-3≤ δ. If $\delta = n-1$ then G is isomorphic to K_n-M where M is a matching in K_n, Then Tγ(G)=3 or 4 then n = 3 or 4 . Since n=3 is impossible, We have n=4 then G is either K_4-e or C_4 for these to graphs Tγ(G)=2≠ $n-3$, Which is a contradiction. Hence $\delta = n-3$. Let X be the vertex cut of G with $|X| = n-3$ and V-X={x_1,x_2,x_3}, X={v_1,v_2,……,v_{n-3}}

Subcase 3.1. $\langle V-X\rangle = \overline{K_3}$

Then every vertex of V-X is adjacent to all the vertices of X then {(V-X)∪$\{v_1\}\} - M$ is a Triple dominating set of G where M is a matching in K_n and hence Tγ(G)≤4 this gives n≤5 , since n≤3 is impossible, We have n = 4 or 5. If n=4 then G is isomorphic to $K_{1,3}$, Which is a contradiction .If n=5 then the graph G has Tγ(G)=3 or 4 which is a contradiction to κ(G) = n – 3

Subcase 3.2. $\langle V-X\rangle = K_1 \cup K_2$

Let $x_1x_2 \in E(G)$ then x_3 is adjacent to all the vertices in X and x_1,x_2 are not adjacent to at most one vertex in X. If $v_1 \notin N(x_1) \cup N(x_2)$ then {(V-X)∪ $\{v_1\}$} is a Triple dominating set of G and hence Tγ(G)≤ 4, This gives n=5 ,For this graph κ(G)=2 which is a contradiction. So all v_i, either $v_i \in N(x_1)$ or $v_i \in N(x_2)$ or both Then{(V-X)∪ $\{v_1\}$} is a Triple dominating set of G by fig:3a. Hence Tγ(G)≤ 4 and then n=4 or 5. If n=4 is impossible , We have n =5, Then G is isomorphic to C_5 or $C_3(P_3,0,0)$. But κ$[C_3(P_3,0,0)]$=1≠n-3 by fig:4c. Hence G is isomorphic to C_5.

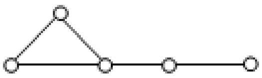
fig:4c

Case 4. Tγ(G) = n and κ(G) = n – 4

Then n-4≤ δ. If δ= n-1 then G is a complete graph which is a contradiction. If δ= n-2 then G is isomorphic to K_n-M where M is a matching in K_n. Then Tγ(G)=3 or 4 then n= 3 or 4 which is a contradiction to κ(G) = n – 4. Suppose δ=n-3. Let X be the vertex cut of G with $|X| = n-4$ and let X={v_1,v_2,……..,v_{n-4}}, V-X = {x_1,x_2,x_3,x_4}. If $\langle V-X\rangle$ contains an isolated vertex then $\delta \le n-4$. Which is a contradiction. Hence $\langle V-X\rangle$ is isomorphic to $K_2 \cup K_2$. Also every vertex of V-X is adjacent to all the vertices of X . Let x_1x_2, $x_3x_4 \in E(G)$. Then {x_1,x_2,x_3,v_1} is a triple dominating set of G. Then Tγ(G)≤4. Hence n≤4, Which is a contradiction to κ(G) = n – 4 . Thus $\delta(G)$ =n-4.

Subcase 4.1. $\langle V-X\rangle = \overline{K_4}$

Then every vertex of V-X is adjacent to all the vertices in X. Suppose $E(\langle X\rangle) = \emptyset$. Then $|X|$ ≤4 and hence G is isomorphic to $K_{s,4}$ where S=1,2,3,4. If S≠ 1,2 then Tγ(G)=3 or 4 which is a contradiction to Tγ(G)= n. Hence G is isomorphic to $K_{3,4}$ or $K_{4,4}$. Suppose $E(\langle X\rangle) = \emptyset$. If any one of the vertex in X say v_i is adjacent to all the vertices in X and hence Tγ(G) ≤3which gives n≤3 which is a contradiction. Hence every vertex in X is not adjacent to at least one vertices in X then {v_1,v_2,v_3,v_4} is a triple dominating set of G and hence Tγ(G) ≤4 then n≤4. Which is a contradiction to κ(G) = n – 4.

Subcase 4.2. $\langle V-X\rangle = P_3 \cup K_1$

Let x_1 be the isolated vertex in $\langle V-X\rangle$ and (x_1,x_2,x_3) be a path then x_1 is adjacent to all the vertices in X and x_2, x_4 are not adjacent to at most two vertices in X and hence {x_1.x_5, x_2.v_1, v_2.x_5, v_1.v_2, x_3.v_1} where $v_1 \in N(x_1) \cap X$, $v_2 \in N(x_2) \cap X$ and $v_3 \in N(x_3) \cap X$ is a triple dominating set of G and hence Tγ(G) ≤5 thus n=5 then G is isomorphic to P_5 or $C_4(1,0,0)$ or $K_3(1,1,0)$ or $(K_4$-e)(1,0,0,0), All these graph Tγ(G)≠n by fig:5a. Which is a contradiction.

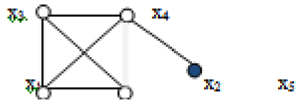

fig:5a

Subcase 4.3. $\langle V-X\rangle = K_3 \cup K_1$

Let x_1 be the isolated vertex in $\langle V-X\rangle$ and $\langle\{x_2, x_3, x_4\}\rangle$ be a complete graph, Then x_1 is adjacent to all the vertex in X and x_2, x_3 ,x_4 are not adjacent to at most two vertices in X and hence {x_2,x_3,x_4,x_5,v_1.x_1} where v_1 ,$v_2 \in$ X-N($x_2 \cup x_3$) is a Triple dominating set of G and hence n=5 by fig:5b. All these graph Tγ(G)≠n.

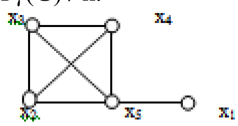

fig.5b

Subcase 4.4. $\langle V-X\rangle = K_2 \cup K_2$

Let x_1x_2, $x_3x_4 \in E(G)$. Since δ(G)=n-4 each x_i is non adjacent to at most one vertex in X then at most one vertex say $v_1 \in X$ such that $|N(v_1) \cap (V-X)|$=1. If all $v_i \in X$ such that $|N(v_i) \cap (V-X)|$ ≥3 then {x_1.v_1,x_2,x_3,x_4} is a triple dominating set of G and hence n=4 . Which is a contradiction by fig:6a. Then each x_i is non adjacent to at most one vertex in X then at most one vertex say v_1 or $v_2 \in X$ such that $|N(v_1) \cap (V-X)|$=2and $|N(v_2) \cap (V-X)|$=2and $|N(v_i) \cap (V-X)|$ ≥3, i≠1 then {v_1.v_2, x_1,x_2,x_3,x_4} is a triple dominating set of G and hence n=5, By fig:6b. Which is a contradiction to $|N(v_1) \cap$ *(V−X)*=2 and *N(v2)∩(V−X)*=2 and $|N(v_i) \cap (V-X)|$ ≥3. The converse is obvious.

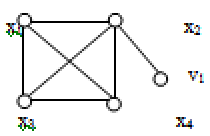

fig:6a

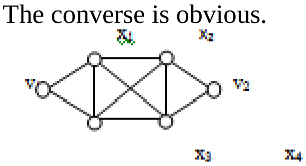

fig:6b

III. CONCLUSION

In this paper we found an upper bound for the sum of Triple domination number and connectivity of graphs and characterized the corresponding extremal graphs. Similarly Triple domination number with other graph theoretical parameters can be considered.

REFERENCES

[1] G. Chartrand and Lesniak, Graphs and Digraphs, CRC, (2005).
[2] T . W. Haynes, S.T.Hedetniemi and P . J. Slater, Fundamentals of Domination in Graphs, Marcel Dekkar, Inc., New York, (1997).
[3] T .W. Haynes, S. T. Hedetniemi and P .J. Slater, Domination in Graphs- Advanced Topics, Marcel Dekker, Inc., New York, (1997).
[4] F .Harary and T . W. Haynes , Double Domination in graphs, Ars combin. 55, 201 - 213(2000).
[5] J. Paulraj Joseph and S .Arumugam, Domination and connectivity in Graphs, International Journal of management and systems, 8, 233-236(1992).
[6] C.Sivagnanam, Double Domination Number and Connectivity of Graphs, International Journal of Digital Information and Wireless Communications (IJDIWC) 2(1):40-45, The Society of Digital Information and Wireless Communications, 2012(ISSN 2225-658X).
[7] J. A.Bondy an U. S. R. Murty, Graph Theory, Springer, 2008.
[8] E. Sampathkumar and, H. B. Walikar, The connected domination number of a graph, J.Math. Phy. Sci., 13(6)(1979), 607-613.
[9] B.D.Acharya, H.B.Walikar, and E.Sampathkumar, Recent developments in the theory of domination in graphs. In MRI Lecture Notes in Math. Mehta Research Instit, Allahabad No.1, (1979).
[10] Teresa L. Tacbobo and Ferdinand P.Jamil, Closed Domination in Graphs, International Mathematical Forum, Vol. 7, 2012, No. 51, 2509-2518.
[11] O. Ore, Theory of graphs , Amer. Math. Soc. Colloq. Publ., Vol.38, Providence, 1962.
[12] C. Berge, Theory of Graphs and its Applications, Methuen, London, 1962.
[13] T. Haynes, S. Hedetniemi and M. Henning , Domination in graphs applied to electrical power networks, J. Discrete Math. 15(4), 519-529, 2000.
[14] A.Nellai Murugan, G.Victor Emmanuel, Degree Equitable Domination Number and Independent Domination Number of a Graph, International Journal of Innovative Research in Science, Engineering and Technology,Vol.2,Issue 11,November 2013 (ISSN: 2319 - 8753)(Accepted).